## WHAT DO YOU NEED TO KNOW TO SOLVE THE TILE PROBLEM?

Step 3: Solution Planning and Monitoring
Strategic Knowledge - need general strategies that can be used to devise and monitor a solution plan

For the tile problem:

- draw a picture

- work backwards from goal: goal is to find total cost of tiling floor, so you need to know the \# of tiles that cover the floor
- divide into subgoals: change dimensions into \# of tiles, then determine how many tiles cover the floor, then determine the cost of all the tiles
*** general strategies are italicized


## KEY QUESTION IN SOLUTION PLANNING: DO I KNOW A RELATED PROBLEM?

Find the area of the figure below:

b

## Givens:

the height and base of the figure

Do I know a related problem?
I know how to find the area of a rectangle. Perhaps I can transform the parallelogram into a rectangle?


## Insight:

All parallelograms can be changed into rectangles
How many lbs of peanuts
(\$1.65 per lb) and almonds
(\$2.10) do you need to
produce a 30 lbs mixture
worth \$1.83 a lb?
$X=$ lbs of peanuts
$Y=$ lbs of almonds
$X+Y=30 \quad X=30-Y$
$X(165)+Y(210)=\mathbf{3 0 ( 1 8 3 )}$
$(\mathbf{3 0}-\mathrm{Y}) \mathbf{1 6 5}+\mathbf{2 1 0 Y}=\mathbf{5 4 9 0}$
$\mathbf{4 9 5 0} \mathbf{- 1 6 5 Y}+\mathbf{2 1 0 Y}=\mathbf{5 4 9 0}$
$\mathbf{4 5 Y}=\mathbf{5 4 0} \quad Y=\mathbf{1 2} \quad X=18$

How many lbs of peanuts (\$1.65 per lb) and almonds (\$2.10) do you need to produce a 30 lbs mixture worth $\$ 1.83$ a lb?
$X=$ lbs of peanuts
$\mathrm{Y}=\mathrm{lbs}$ of almonds
$X+Y=30 \quad X=30-Y$
$X(165)+Y(210)=30(183)$
$(\mathbf{3 0}-\mathrm{Y}) \mathbf{1 6 5}+\mathbf{2 1 0 Y}=5490$
$4950-165 Y+210 Y=5490$
$45 Y=540 \quad Y=12 \quad X=18$

How much of a 6\% boric acid solution and a $12 \%$ boric acid solution do you need to produce 4.5 pints of a 8\% solution?
$X=$ pints of $6 \%$ solution
$Y=$ pints of $12 \%$ solution
$X+Y=4.5 \quad X=4.5-Y$
$X(6)+Y(12)=4.5(8)$
$(4.5-Y) 6+12 Y=36$
$27-6 Y+12 Y=36$
$6 Y=9 \quad Y=1.5 \quad X=3$

## FAULTY BELIEFS ABOUT MATH THAT UNDERMINE EFFECTIVE PROBLEM SOLVING

1) Ordinary students cannot expect to understand math, they have to memorize it, and just apply what they have learned mechanically and without understanding.
2) All story problems can be solved by applying operations suggested by key words in the story (in all suggests addition, left suggests subtraction, share suggests division - 3rd graders)
3) Any assigned problem should be solved within five minutes or less. (High school students estimated the typical problem should take about 2 minutes)
4) Math is not particularly useful or sensible. Math is mostly a set of rules and mathematics learning means memorizing the rules (54\% of 4th graders and $\mathbf{4 0 \%}$ of eighth graders; females' attitudes toward math more negative).

## ATTRIBTUTION STYLE UNDERMINES EFFECTIVE MATH PROBLEM SOLVING

Researchers gave 10 year old children a questionnaire asking about their likely reactions to hypothetical failures.

They identified two attribution styles:
Mastery-oriented: likely to think they should work harder in the face of failure/difficulty

Helpless: likely to respond to difficulty with negative attributions about ability
*** There were no IQ differences between these two groups.
*** Many more girls were categorized as "helpless."

## ATTRIBTUTION STYLE UNDERMINES EFFECTIVE MATH PROBLEM SOLVING - CONTINUED

Researchers next gave the children a series of confusing math problems (difficult to solve), and then a batch of easy math problems (that all children should be able to solve).

What happened?
Mastery-oriented children: These children were able to recoup from the negative experience and solved the easy problems with ease.

Helpless: These children were thrown by the confusing problems and didn't try very hard on the easy problems, getting many of them wrong.

## ATTRIBTUTION STYLE UNDERMINES EFFECTIVE MATH PROBLEM SOLVING - CONTINUED

Researchers wanted to know why girls were more likely to adopt a "helpless" attribution style.

What happens in the classroom?
Boys and girls receive the same amount of negative comments. But the nature of these comments differ.

Boys: Criticisms sometimes focus on intellectual quality, sometimes on neatness, conduct, or effort. Boys and girls both think teachers like girls better.

Girls: Teacher criticisms focus consistently on the intellectual quality of the work.

End Result: Boys attribute failure to any number of factors, girls are left with negative attributions concerning their ability.

## WHAT DO YOU NEED TO KNOW TO SOLVE THE TILE PROBLEM?

Step 4: Solution Execution
Procedural Knowledge - computational procedures from simple procedures (e.g., single digit addition or subtraction) to more complex procedures (e.g., subtraction of multiple digit numbers)

For the tile problem:

$$
\begin{aligned}
& 540 / 30=18 \text { tiles } \quad 720 / 30=24 \text { tiles } 18 \times 24=432 \text { tiles } \\
& 432 \text { times } \$ .72=\$ 311.04
\end{aligned}
$$

*** Key Point: able to do computations with no difficulty, fast and accurate (achieve automaticity, direct retrieval from longterm memory)

## THE TRANSITION TO AUTOMATICITY: SIMPLE ADDITION

Step 1: Count 1st number and then the 2nd on your fingers
Step 2: Move on to the most efficient counting strategy, the "min" strategy
e.g., $2+4$
raise four fingers and then count up by 2
Step 3: Be strategic
when you get a problem, retrieve an answer from long term memory
if you're not sure, use your counting strategy or use some known facts (I know $\mathbf{2}+\mathbf{2}$ is $\mathbf{4}$, so $\mathbf{2}+\mathbf{3}$ must be 5 )
*** by selecting the best strategy for coming up with the correct answer, children move toward automaticity more quickly

