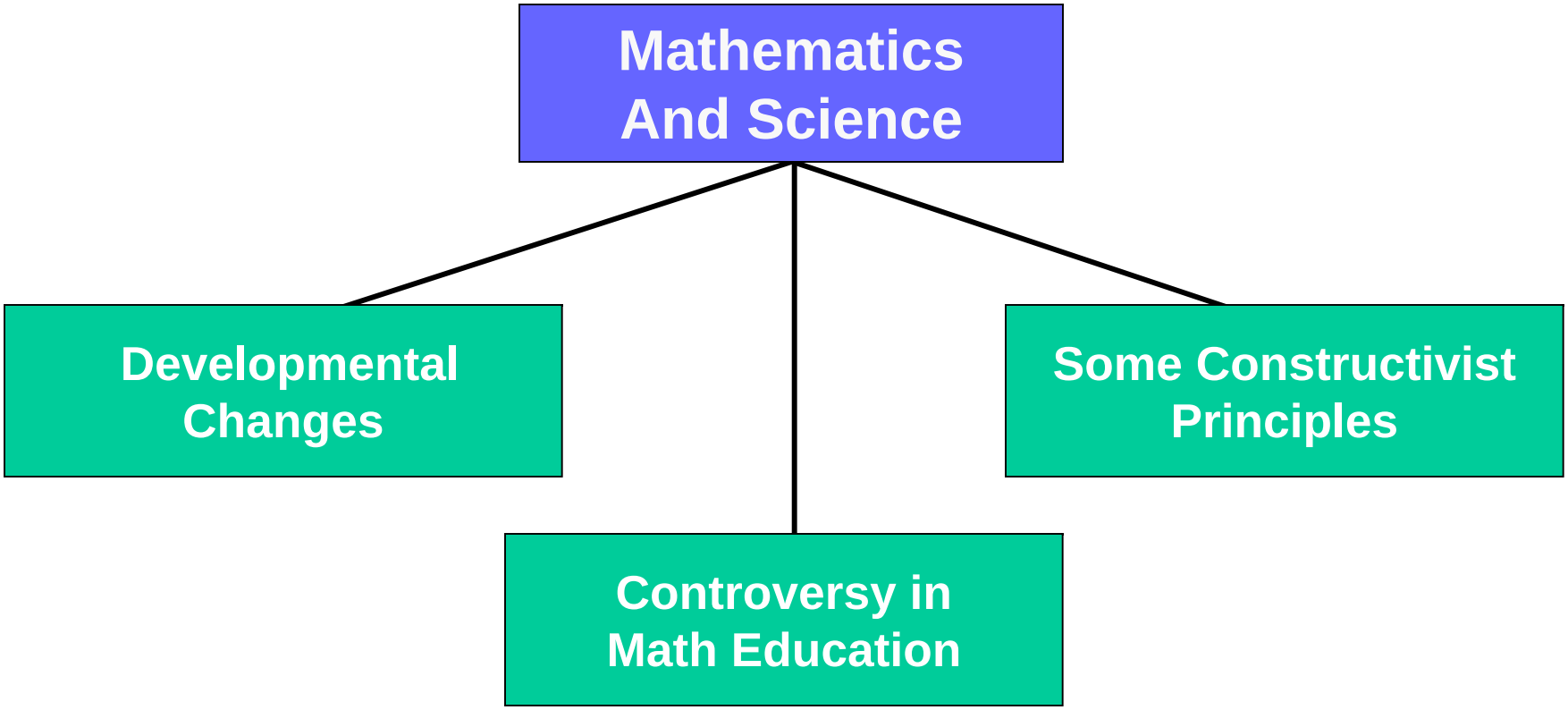
Learning and Cognition in the Content Areas



Developmental Changes in Mathematics

Pre-K–Grade 2 Count past 20, add/subtract single digits

Grades 3–5 Multiplicative reasoning, equivalence, and computational fluency

Developmental Changes in Mathematics

Grades 6–8 Algebra and geometry, quantitative life solutions

Grades 9–12 Students should experience algebra, geometry, statistics, probability, and discrete mathematics

TEACHING MATH - TWO TRADITIONS

DRILL & PRACTICE

2 + 3 = ?

4 - 2 = ?

5 - 3 = ?

4 + 7 = ?

9 + 7 = ?

12
+ 24

32
- 17

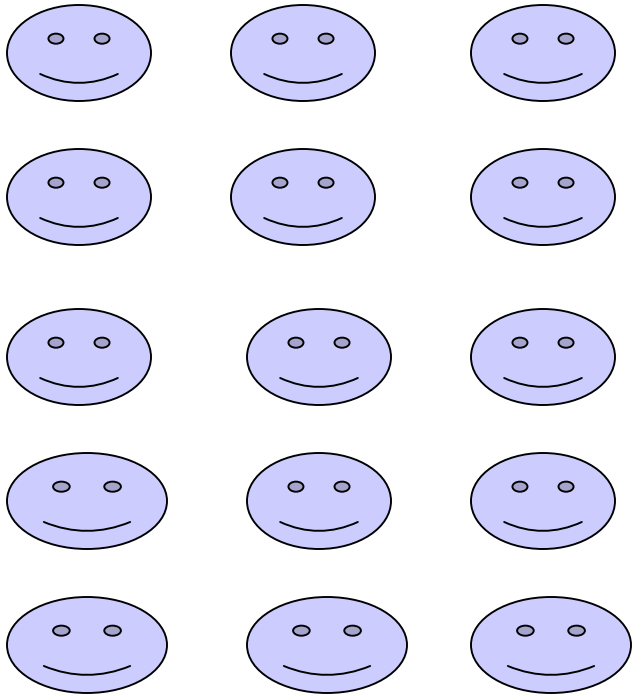
5 x 6 = ?

2 + ? = 7

5 x ? = 45

MEANINGFUL APPROACH

3 x 5 = ?



PRINCIPLES OF DRILL AND PRACTICE

- **SIMPLE TO COMPLEX (order of problem presentation)**

- **DISTRIBUTED PRACTICE BETTER (e.g., if you are going to study for one hour, break it down into two 1/2 hour or 3 twenty minutes segments)**

- **MIXED PRACTICE BETTER (at some point, when the student had some initial practice with individual problem types)**

Associative Strengths for Simple Addition Problems 4-5 yr olds

Problem	Answer												
	0	1	2	3	4	5	6	7	8	9	10	11	other
1 + 1		.05	.86		.02		.02					.02	.04
1 + 4					.11	.61	.09	.07				.02	.11
1 + 5					.13	.16	.50	.11		.02	.02		.05
2 + 2	.02		.04	.05	.80	.04		.05					
2 + 3			.04	.07	.38	.34	.09	.02	.02	.02			.04
2 + 4		.02		.07	.02	.43	.29	.07	.07				.04
4 + 1			.04	.02	.09	.68	.02	.02	.07				.07
5 + 1			.04		.04	.07	.71	.04	.04		.04		.04
5 + 4					.11	.21	.16	.05	.11	.16	.04		.16

PRINCIPLES OF THE MEANINGFUL APPROACH

- **USE CONCRETE EXAMPLES, IMAGERY**

(to support understanding)

- **WHEN A STUDENT HAS DIFFICULTY,**

DIAGNOSE CONCEPUTAL CONFUSION

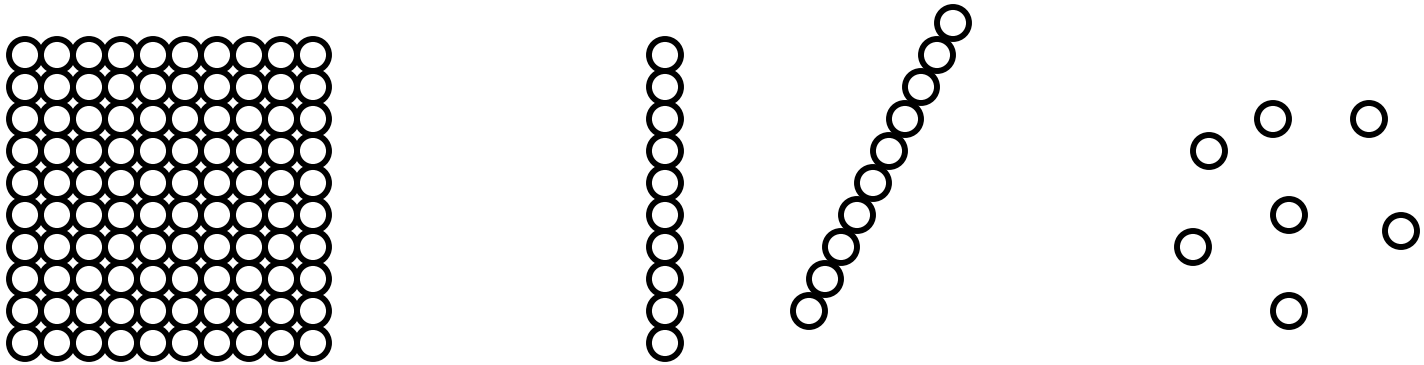
- **BUILD UP UNDERSTANDING BY RELYING ON**

VISUAL AND ENACTIVE REPRESENTATION

(see examples in the next few slides)

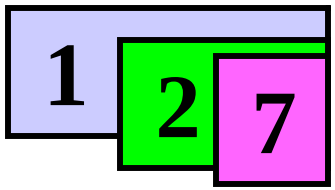
TEACHING POSITIONAL NOTATION

Array of Beads - Concrete Representation



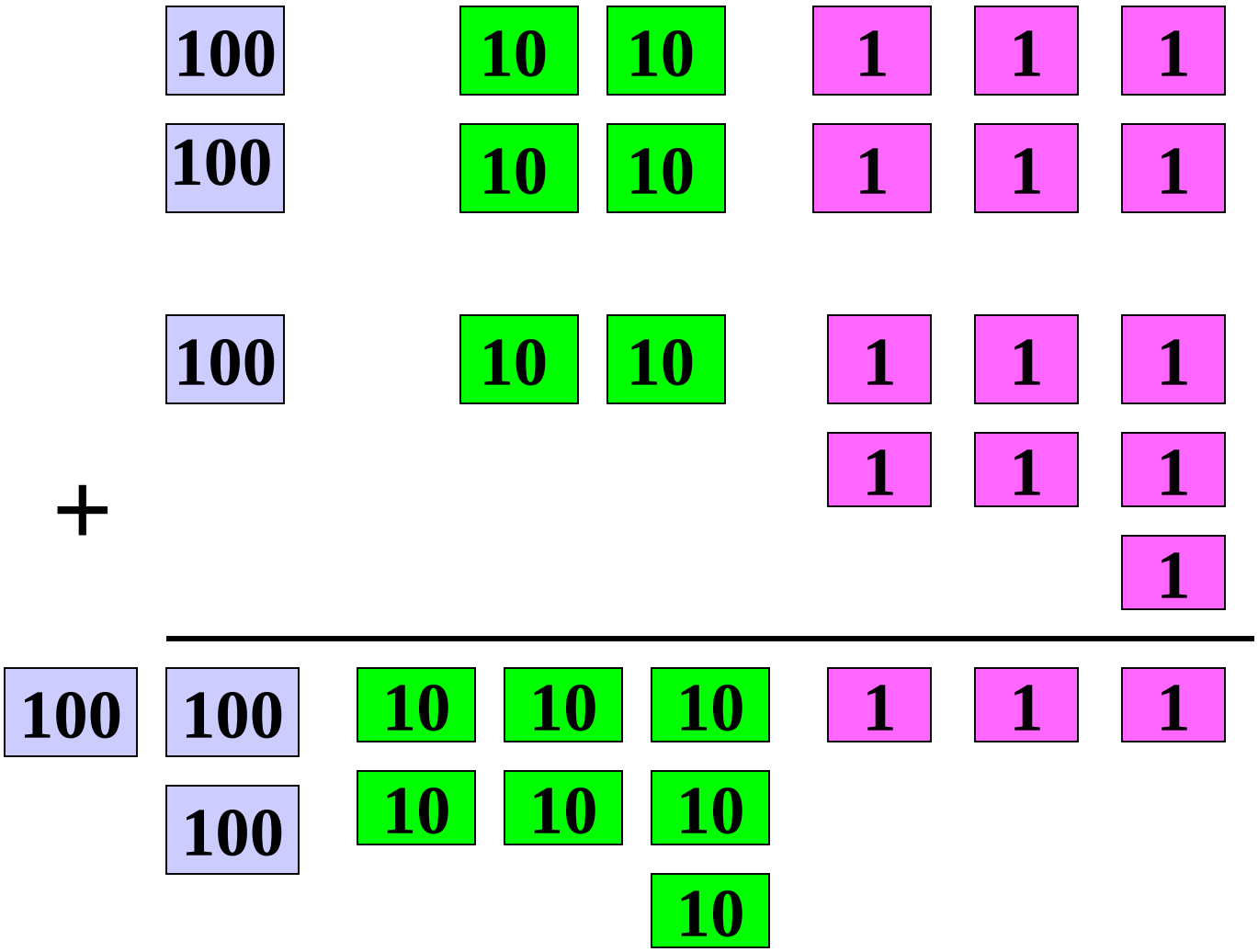
Color Coded Labels - Intermediate Step

Superimposed Labels



ADDITION WITH CARRYING

Color Coded Squares - Intermediate Step



Short cut Algorithm

```
  1
  2 4 6
+ 1 2 7
-------
  3 7 3
```

SAMPLE MATH "BUGS"

I. Addition - *What's wrong???*

23	18	45
+ 14	+ 31	+ 22
73	94	76

II. Subtraction - *What's wrong???*

23	34	41
– 14	– 16	– 22
11	22	21

MORE SAMPLE MATH "BUGS"

III. Rounding off - *What's Wrong???*

45.254 (to nearest tenth) 45.2

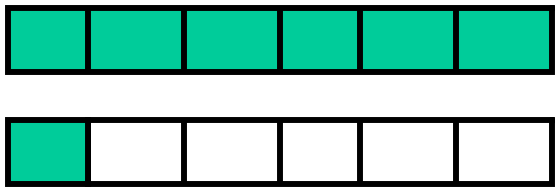
156.327 (to nearest tenth) 156.4

45.254 (to nearest hundredth) 45.25

156.327 (to nearest hundredth) 156.33

IV. Understanding Fractions (represent the shaded bars as a mixed number & a fraction) - *What's Wrong???*

1 1/3 4/6



1 1/6 7/12

Complex Arithmetic

The Need for Genuine Understanding!

Problem: Estimate the answer to 12/13 and 7/8. You will not have time to solve the problem with paper and pencil.

Answer	Age 13	Age 17
1	7%	8%
2	24%	37%
19	28%	21%
21	27%	15%
Don’t Know	14%	16%

Math Problem Solving

The Need for Genuine Understanding!
And Representational Fluency!

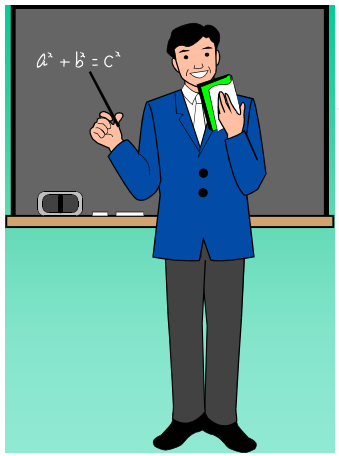
Problem: "There are six times as many students as professors at this university."

6s = p vs. 6p = s

37% of freshmen engineering students at a major state university could not write the correct equation *representing* the situation.

Constructivist Principles for Teaching Mathematics

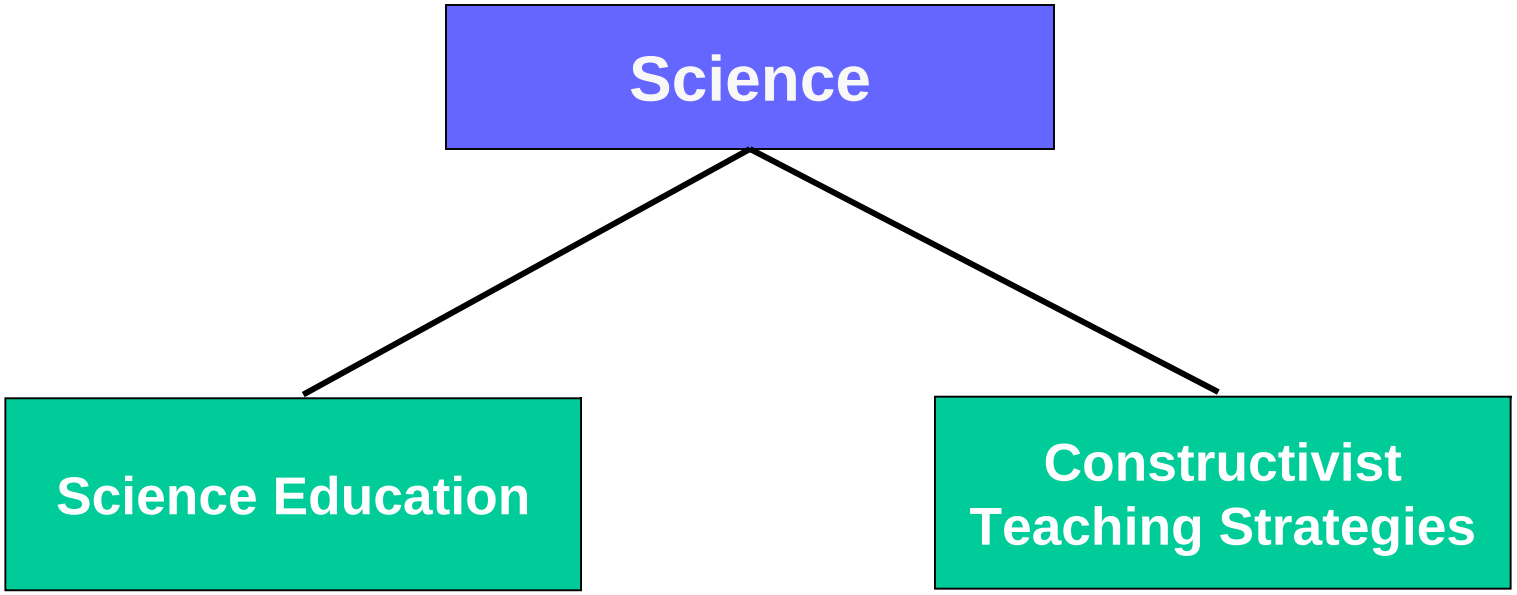
Make math realistic and interesting



With appropriate prior knowledge, make students stretch their minds to solve problems

Make math curriculum socially interactive

Learning and Cognition in the Content Areas



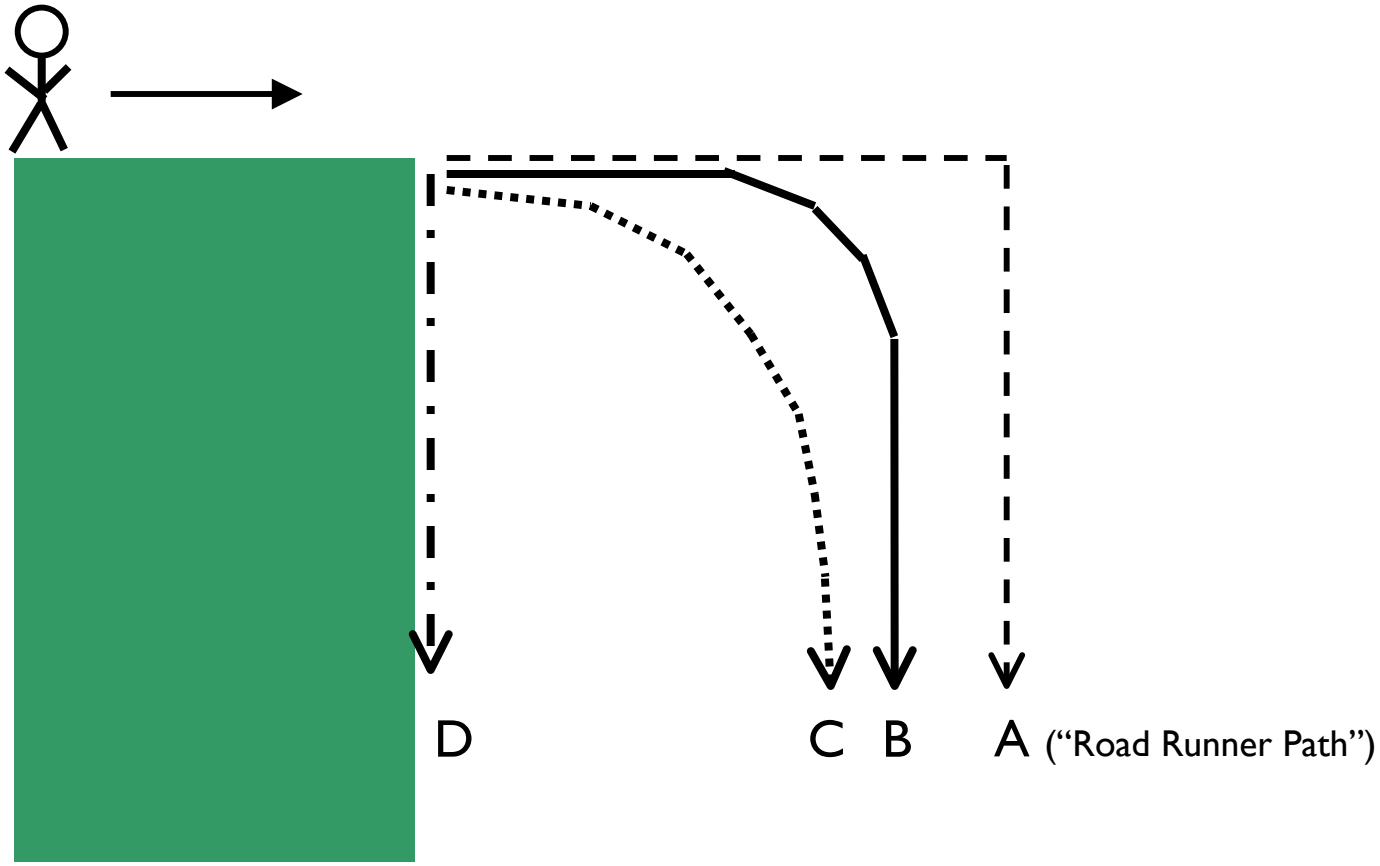
How Do Scientists Think & Behave?

- They make careful observations
- They collect, organize and analyze data
- They measure, graph, and understand spatial relations
- They pay attention to and regulate their own thinking
- They know when and how to apply their knowledge to solve problems

Teacher Challenge: Student Misconceptions

Problem: Moving object over a cliff

Consider a typical cartoon character who runs over a cliff and falls into the valley below. With a pencil, you should draw the path that the falling body will follow. *Subjects' responses in black (A,B,C,D).*



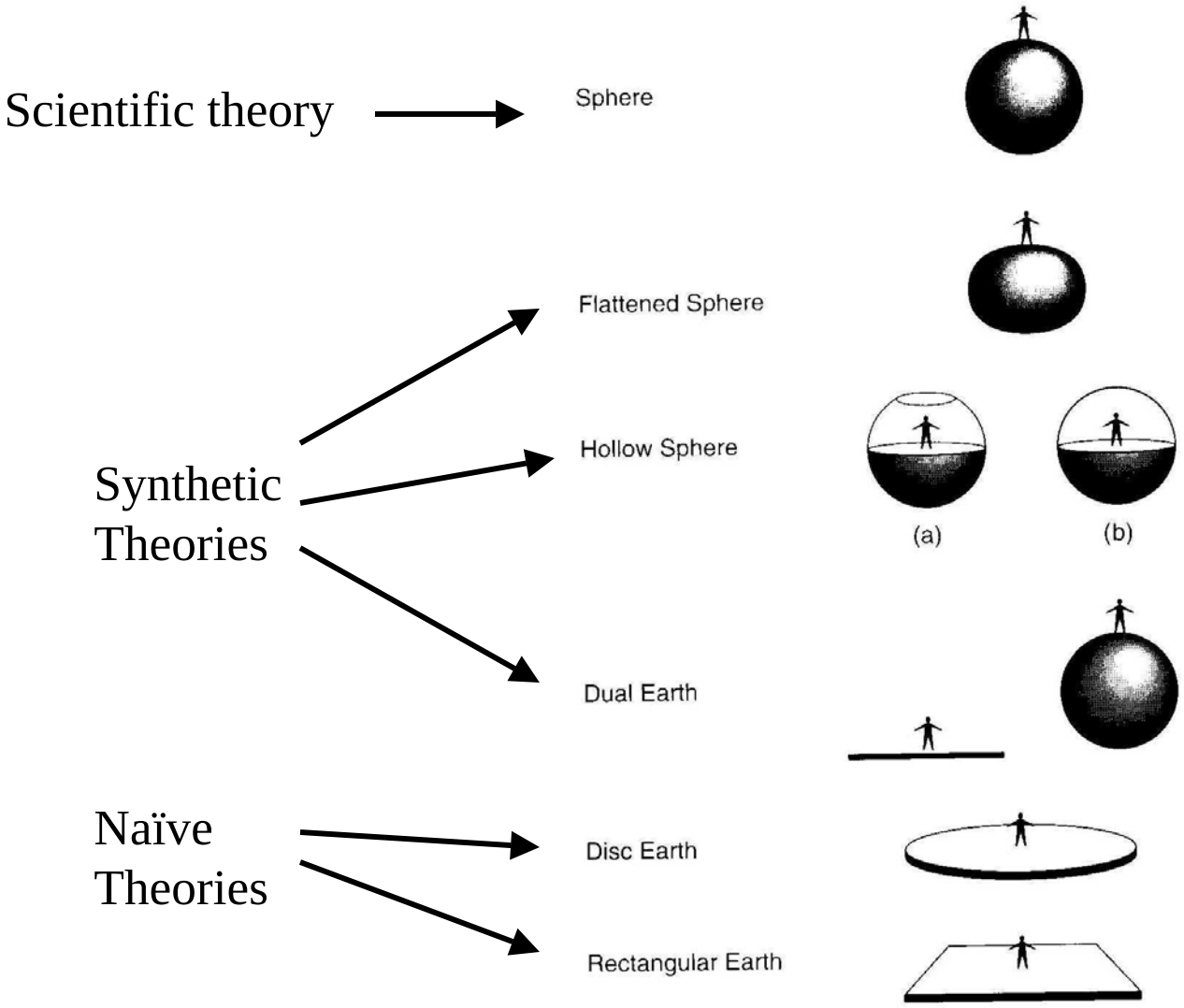
CONFRONTING STUDENTS' MISCONCEPTIONS

Problem: Moving object over a cliff

Answers from High School and College Students

Path A	**5%**	**It will go straight for a distance and then go straight down *(roadrunner path)***
Path B	**35%**	**It will go straight for a distance and then gradually arc down *(impetus theory)***
Path C	**28%**	**It will arc downward, maintaining constant forward speed and increasing downward speed *(correct)***
Path D	**32%**	**It will fall straight down as soon as it leaves the edge of the cliff**

Teacher Challenge: Student Misconceptions

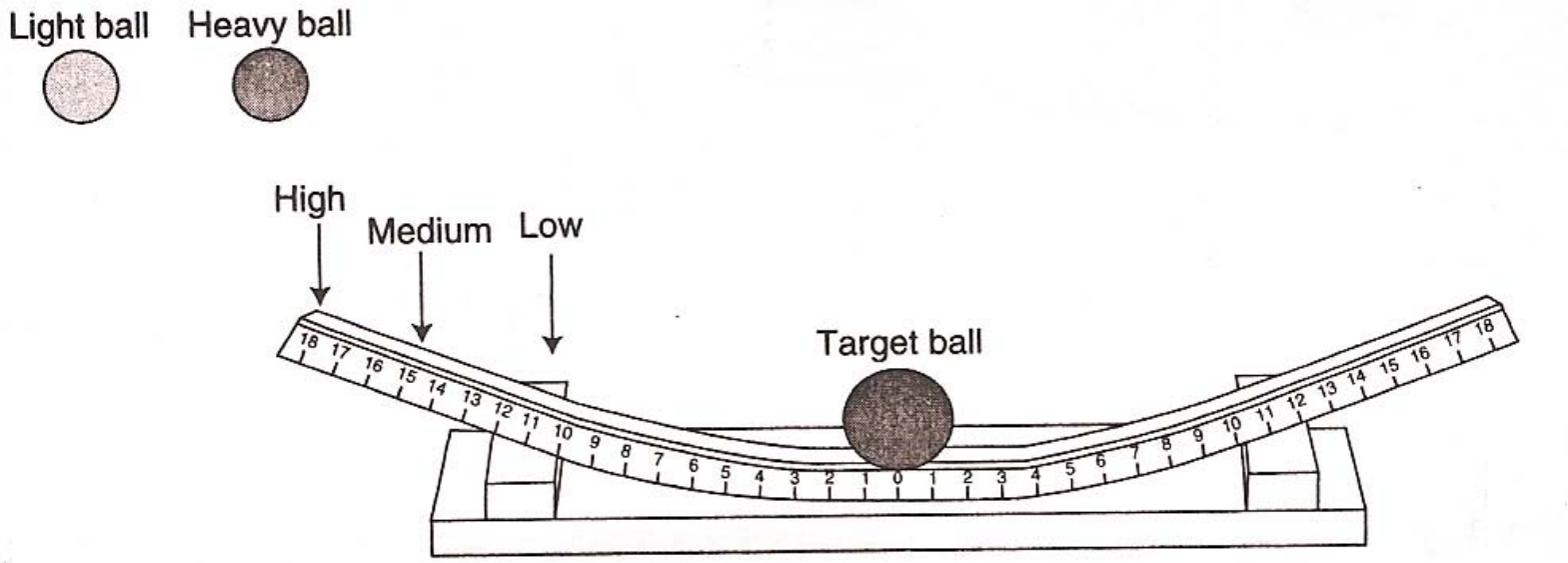


6-11yr olds

Even for 5th graders, 40% had mental models other than the sphere model

Teacher Challenge: Control of Variables Task

Cross-Cultural Study – 7 Western Countries – 13 to 15 yr olds



14% of the students have mastered systematic testing of the effects of each variable – one at a time

Strategies for Teaching Science

- Help students learn how to think like scientists
- Monitor students' misconceptions about science and working with them to develop more accurate conceptions
- Guide students in developing inquiry skills
- Teach science content as well
- Make science interesting by giving students opportunities to explore everyday science problems